

### Hw 3

2.2.2 page 93

2.2.4 page 93

2.2.9 page 94

2.3.3 page 94

2.3.7 page 94

2.3.11 page 95

**2.2.2**

For random variables  $X$  and  $R$  defined in Example 2.5, find  $P_X(x)$  and  $P_R(r)$ . In addition, find the following probabilities:

- (a)  $P[X = 0]$
- (b)  $P[X < 3]$
- (c)  $P[R > 1]$

**Example 2.5**

Suppose we observe three calls at a telephone switch where voice calls ( $v$ ) and data calls ( $d$ ) are equally likely. Let  $X$  denote the number of voice calls,  $Y$  the number of data calls, and let  $R = XY$ . The sample space of the experiment and the corresponding values of the random variables  $X$ ,  $Y$ , and  $R$  are

Outcomes		$ddd$	$ddv$	$dvd$	$dvv$	$vdd$	$vdv$	$vvd$	$vvv$
$P[\cdot]$		$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$	$1/8$
Random Variables	$X$	0	1	1	2	1	2	2	3
	$Y$	3	2	2	1	2	1	1	0
	$R$	0	2	2	2	2	2	2	0

**2.2.4**

The random variable  $X$  has PMF

$$P_X(x) = \begin{cases} c/x & x = 2, 4, 8, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) What is the value of the constant  $c$ ?
- (b) What is  $P[X = 4]$ ?
- (c) What is  $P[X < 4]$ ?
- (d) What is  $P[3 \leq X \leq 9]$ ?

2.2.9

When someone presses "SEND" on a cellular phone, the phone attempts to set up a call by transmitting a "SETUP" message to a nearby base station. The phone waits for a response and if none arrives within 0.5 seconds it tries again. If it doesn't get a response after  $n = 6$  tries the phone stops transmitting messages and generates a busy signal.

- (a) Draw a tree diagram that describes the call setup procedure.
- (b) If all transmissions are independent and the probability is  $p$  that a "SETUP" message will get through, what is the PMF of  $K$ , the number of messages transmitted in a call attempt?
- (c) What is the probability that the phone will generate a busy signal?
- (d) As manager of a cellular phone system, you want the probability of a busy signal to be less than 0.02. If  $p = 0.9$ , what is the minimum value of

2.3.3

When you go fishing, you attach  $m$  hooks to your line. Each time you cast your line, each hook will be swallowed by a fish with probability  $h$ , independent of whether any other hook is swallowed. What is the PMF of  $K$ , the number of fish that are hooked on a single cast of the line?

**2.3.7**

The number of buses that arrive at a bus stop in  $T$  minutes is a Poisson random variable  $B$  with expected value  $T/5$ .

- (a) What is the PMF of  $B$ , the number of buses that arrive in  $T$  minutes?
- (b) What is the probability that in a two-minute interval, three buses will arrive?
- (c) What is the probability of no buses arriving in a 10-minute interval?
- (d) How much time should you allow so that with probability 0.99 at least one bus arrives?

**2.3.11**

In a packet voice communications system, a source transmits packets containing digitized speech to a receiver. Because transmission errors occasionally occur, an acknowledgment (ACK) or a nonacknowledgment (NAK) is transmitted back to the source to indicate the status of each received packet. When the transmitter gets a NAK, the packet is retransmitted. Voice packets are delay sensitive and a packet can be transmitted a maximum of  $d$  times. If a packet transmission is an independent Bernoulli trial with success probability  $p$ , what is the PMF of  $T$ , the number of times a packet is transmitted?